

Lepton-flavor-violating Higgs decay $h \rightarrow \mu\tau$ and muon anomalous magnetic moment in a general two Higgs doublet model

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A two Higgs doublet model (2HDM) is one of minimal extensions of the Standard Model (SM), and it is well-known that the general setup predicts the flavor-violating phenomena, mediated by neutral Higgs interactions. Recently the CMS collaboration has reported an excess of the lepton-flavor-violating Higgs decay in $h \rightarrow \mu\tau$ channel with a significance of 2.4σ . We investigate the CMS excess in a general 2HDM with tree-level Flavor Changing Neutral Currents (FCNCs), and discuss its impact on the other physical observations. Especially, we see that the FCNCs relevant to the excess can enhance the neutral Higgs contributions to the muon anomalous magnetic moment, and can resolve the discrepancy between the measured value and the SM prediction. We also find that the couplings to be consistent with the anomaly of the muon magnetic moment as well as the CMS excess in $h \rightarrow \mu\tau$ predict the sizable rate of $\tau \rightarrow \mu\gamma$, which is within the reach of future B factory.

While a Higgs boson has been discovered at the Large Hadron Collider (LHC) experiment [1, 2], the whole structure of the Higgs sector is still unknown. Theoretically there is no apparent reason why a Higgs sector with one Higgs doublet is better than the one with more Higgs doublets. Thus, only the experimental research will reveal the true answer.

A two Higgs doublet model (2HDM) is a simple extension of the minimal Higgs sector in the SM. In general, both Higgs doublets couple to fermions, and hence the flavor-changing Higgs interaction is predicted. This is one of the main differences from the SM. Recently the CMS collaboration has reported an excess of lepton-flavor-violating Higgs decay in $h \rightarrow \mu\tau$ mode [3, 4]. The SM cannot accommodate such an excess, however, the general 2HDM¹ can explain the excess, as pointed out in Refs. [5–7].² Therefore, it is worth studying it further, and we find that the $\mu - \tau$ lepton-flavor-violating Higgs interaction can enhance the neutral Higgs contributions to an anomalous magnetic moment of muon (muon g-2), and hence it can explain the long-standing anomaly of the muon g-2 [15].

In the general 2HDM, we can always take a basis where only one Higgs doublet gets a vacuum expectation value (VEV), so that we can parametrize the Higgs doublets as follows;

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+\phi_1+iG}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+iA}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

where G^+ and G are Nambu-Goldstone bosons, and H^+ and A are a charged Higgs boson and a CP-odd Higgs boson, respectively. CP-even neutral Higgs bosons ϕ_1 and ϕ_2 can mix and form mass eigenstates, h and H ($m_H > m_h$),

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\beta\alpha} & \sin\theta_{\beta\alpha} \\ -\sin\theta_{\beta\alpha} & \cos\theta_{\beta\alpha} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}. \quad (2)$$

Here $\theta_{\beta\alpha}$ is the mixing angle. In mass eigenbasis for the fermions, the Yukawa interactions are expressed as follows;

$$\begin{aligned} \mathcal{L} = & -\bar{Q}_L^i H_1 y_d^i d_R^i - \bar{Q}_L^i H_2 \rho_d^{ij} d_R^j \\ & -\bar{Q}_L^i (V_{\text{CKM}}^\dagger)^{ij} \tilde{H}_1 y_u^j u_R^j - \bar{Q}_L^i (V_{\text{CKM}}^\dagger)^{ij} \tilde{H}_2 \rho_u^{jk} u_R^k \\ & -\bar{L}_L^i H_1 y_e^i e_R^i - \bar{L}_L^i H_2 \rho_e^{ij} e_R^j, \end{aligned} \quad (3)$$

where $Q = (V_{\text{CKM}}^\dagger u_L, d_L)^T$, $L = (V_{\text{MNS}} \nu_L, e_L)^T$, $V_{\text{CKM}}(V_{\text{MNS}})$ is the Cabbibo-Kobayashi-Maskawa (Maki-Nakagawa-Sakata) matrix and the fermions (f_L, f_R) ($f = u, d, e, \nu$) are mass eigenstates. ρ_f^{ij} are general 3-by-3 complex matrices and can be sources of the Higgs-mediated FCNC processes. In the following discussions, we do not adopt the so-called Cheng-Sher ansatz [16] for ρ_f^{ij} in order to explore wider parameter space.

¹ Sometimes, it is called the Type III two Higgs doublet model.

² Multi-Higgs doublet model has been also considered [8]. The lepton flavor violating Higgs decays have been investigated before the Higgs discovery [9–14].

In the mass eigenstate of Higgs bosons, the interactions are expressed as

$$\begin{aligned}\mathcal{L} = & - \sum_{\phi=h,H,A} y_{\phi ij} \bar{f}_{Li} \phi f_{Rj} - \bar{\nu}_{Li} (V_{\text{MNS}}^\dagger \rho_e)^{ij} H^+ e_{Rj} \\ & - \bar{u}_i (V_{\text{CKM}} \rho_d P_R - \rho_u^\dagger V_{\text{CKM}} P_L)^{ij} H^+ d_j + \text{h.c.},\end{aligned}\quad (4)$$

where

$$\begin{aligned}y_{hij} &= \frac{m_f^i}{v} s_{\beta\alpha} \delta_{ij} + \frac{\rho_f^{ij}}{\sqrt{2}} c_{\beta\alpha}, \quad y_{Hij} = \frac{m_f^i}{v} c_{\beta\alpha} \delta_{ij} - \frac{\rho_f^{ij}}{\sqrt{2}} s_{\beta\alpha}, \\ y_{Aij} &= -\frac{i\rho_f^{ij}}{\sqrt{2}} \text{ (for } f = u), \quad \frac{i\rho_f^{ij}}{\sqrt{2}} \text{ (for } f = d, e),\end{aligned}\quad (5)$$

and $s_{\beta\alpha} = \sin\theta_{\beta\alpha}$ and $c_{\beta\alpha} = \cos\theta_{\beta\alpha}$ are defined. Note that the SM-like Higgs couplings y_{hff} approach to the SM ones when $c_{\beta\alpha}$ gets closer to zero, so that the flavor-violating phenomena mediated by the SM-like Higgs boson can be suppressed in this limit. The current LHC Higgs coupling measurements and search for flavor violation suggest the smallness of the mixing parameter $c_{\beta\alpha}$ in this framework.

On the other hand, the CMS collaboration reports that there is an excess in $h \rightarrow \mu\tau$ process [3, 4];

$$\text{BR}(h \rightarrow \mu\tau) = (0.84_{-0.37}^{+0.39})\%, \quad (6)$$

where the final state is a sum of $\mu^+\tau^-$ and $\mu^-\tau^+$. This might be an evidence of a Flavor Changing Neutral Current (FCNC) involving SM-like neutral Higgs, and, in fact, the flavor-violating coupling ρ_e can accommodate the CMS result in our general 2HDM;

$$\text{BR}(h \rightarrow \mu\tau) = \frac{c_{\beta\alpha}^2 (|\rho_e^{\mu\tau}|^2 + |\rho_e^{\tau\mu}|^2) m_h}{16\pi\Gamma_h}, \quad (7)$$

where Γ_h is a total decay width of Higgs boson h and we adopt $\Gamma_h = 4.1$ MeV in this paper. In order to explain the excess, the size of the flavor mixing should be as follows;

$$\begin{aligned}\bar{\rho}^{\mu\tau} &\equiv \sqrt{\frac{|\rho_e^{\mu\tau}|^2 + |\rho_e^{\tau\mu}|^2}{2}}, \\ &\simeq 0.26 \left(\frac{0.01}{c_{\beta\alpha}} \right) \sqrt{\frac{\text{BR}(h \rightarrow \mu\tau)}{0.84 \times 10^{-2}}}.\end{aligned}\quad (8)$$

Even if the Higgs mixing is small ($c_{\beta\alpha} = 0.01$), the $O(1)$ flavor-violating coupling $\bar{\rho}^{\mu\tau}$ can achieve the CMS excess.

The next question is what kind of prediction we have, if such a flavor-violating Yukawa coupling exists. One interesting observable predicted by the FCNC is the muon g-2, where the discrepancy between the experimental result and the SM prediction is reported. The CMS excess requires the sizable $\mu - \tau$ flavor violation, so that it would be possible for the large FCNC to contribute to the muon g-2 through the one-loop diagram involving neutral scalars (h, H, A), as we see Fig. 1. The extra contributions from $\rho_e^{\mu\tau (\tau\mu)}$ induce the deviation from the SM prediction;

$$\delta a_\mu \simeq \frac{m_\mu m_\tau \rho_e^{\mu\tau} \rho_e^{\tau\mu}}{16\pi^2} \left[\frac{c_{\beta\alpha}^2 (\log \frac{m_h^2}{m_\tau^2} - \frac{3}{2})}{m_h^2} + \frac{s_{\beta\alpha}^2 (\log \frac{m_H^2}{m_\tau^2} - \frac{3}{2})}{m_H^2} - \frac{\log \frac{m_A^2}{m_\tau^2} - \frac{3}{2}}{m_A^2} \right],$$

assuming that $\rho_e^{\mu\tau} \rho_e^{\tau\mu}$ is real, for simplicity.³ Here we only consider the dominant contributions which are proportional to τ mass m_τ .⁴ We note that the Yukawa couplings $\rho_e^{\mu\tau (\tau\mu)}$ generate an enhancement of $O(m_\tau/m_\mu)$ in the δa_μ , where the m_τ dependence comes from the internal τ lepton propagator in one loop diagram shown in Fig. 1. To maximize a size of the δa_μ , while keeping a value of $\text{BR}(h \rightarrow \mu\tau)$, $|\rho_e^{\mu\tau}| \sim |\rho_e^{\tau\mu}|$ is preferred.

³ If $\rho_e^{\mu\tau} \rho_e^{\tau\mu}$ is complex, the electric dipole moment (EDM) of the muon would be induced. The current limit of the muon EDM is $|d_\mu| < 1.8 \times 10^{-19}$ e cm [17], which is expected to be improved up to 1×10^{-24} e cm in the future experiments [18, 19].

⁴ In general, the other Yukawa couplings ρ_e might contribute to the muon g-2. Here we have simply assumed that the others are negligible.

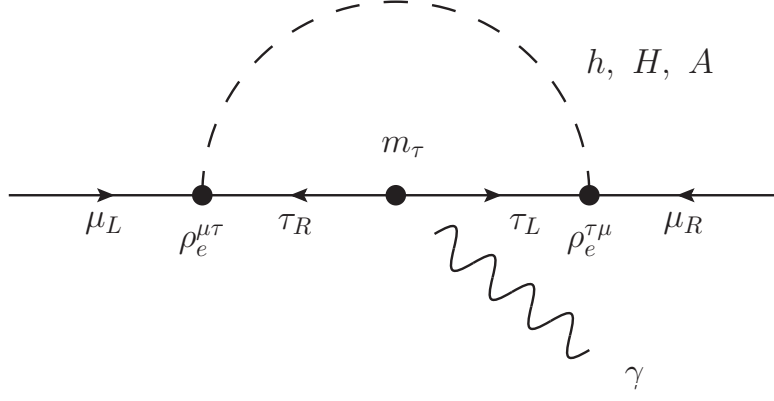


FIG. 1: A Feynman diagram for neutral Higgs boson contributions to the muon g-2. A photon is attached somewhere in the charged lepton line.

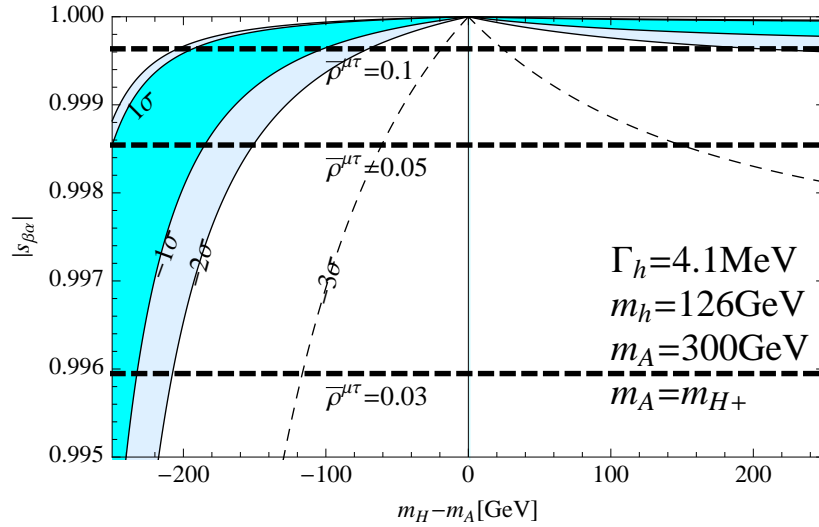


FIG. 2: The neutral Higgs contributions to the muon g-2 (δa_μ) induced by the lepton flavor violating couplings $\rho_e^{\mu\tau(\tau\mu)}$ as functions of $|s_{\beta\alpha}|$ and $m_H - m_A$. Here we assume $\bar{\rho}^{\mu\tau} = \rho_e^{\mu\tau} = \pm \rho_e^{\tau\mu}$ where the sign of the $\rho_e^{\tau\mu}$ is fixed to induce the positive contribution to δa_μ and the value of $\bar{\rho}^{\mu\tau}$ is determined to explain the CMS excess of $\text{BR}(h \rightarrow \mu\tau)$. We have taken $m_A = m_{H^+} = 300$ GeV. The cyan (light blue) region is the one within $|1\sigma|$ ($|2\sigma|$) range for the muon g-2 anomaly with the 1σ uncertainty of the CMS $h \rightarrow \mu\tau$ excess. The dashed is -3σ line. The thick dashed lines correspond to $\bar{\rho}_{\mu\tau} = 0.1, 0.05$ and 0.03 with $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$, respectively.

Fig. 2 shows the numerical result of δa_μ induced by the lepton-flavor-violating couplings $\rho_e^{\mu\tau(\tau\mu)}$ as functions of $|s_{\beta\alpha}|$ and a mass difference between H and A , $m_H - m_A$. Here we have assumed $\bar{\rho}^{\mu\tau} = \rho_e^{\mu\tau} = \pm \rho_e^{\tau\mu}$ where the sign of $\rho_e^{\tau\mu}$ is chosen to realize the positive contribution to δa_μ and the value of $\bar{\rho}^{\mu\tau}$ is determined to explain the CMS excess of $\text{BR}(h \rightarrow \mu\tau)$. We have taken $m_A = m_{H^+} = 300$ GeV. In the cyan (light blue) region of Fig. 2, the anomaly of the muon g-2 can be explained within $|1\sigma|$ ($|2\sigma|$) with the 1σ uncertainty of the CMS $h \rightarrow \mu\tau$ excess. The -3σ line for the muon g-2 anomaly is also shown. Here we adopt the value of the muon g-2 anomaly from Ref. [20], $\delta a_\mu = (26.1 \pm 8.0) \times 10^{-10}$. In Fig. 2, the thick dashed lines correspond to $\bar{\rho}^{\mu\tau} = 0.1, 0.05$ and 0.03 with $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$, respectively.

In order to explain the anomaly of the muon g-2, the Higgs mixing parameter $|s_{\beta\alpha}|$ should be close to one, which is consistent with the current Higgs coupling measurements at the LHC experiment. Note that the non-degeneracy among neutral Higgs bosons induces the larger δa_μ . Although the non-degeneracy also generates the extra contributions to Peskin-Takeuchi's T-parameter [21–24], we have found that the small Higgs mixing parameter $c_{\beta\alpha}$ suppresses the extra contributions in the current scenario when m_A is very close to m_{H^+} .

As pointed out in Refs. [5, 25], the Yukawa couplings $\rho_e^{\mu\tau(\tau\mu)}$ would also induce significant contributions to $\tau \rightarrow \mu\gamma$

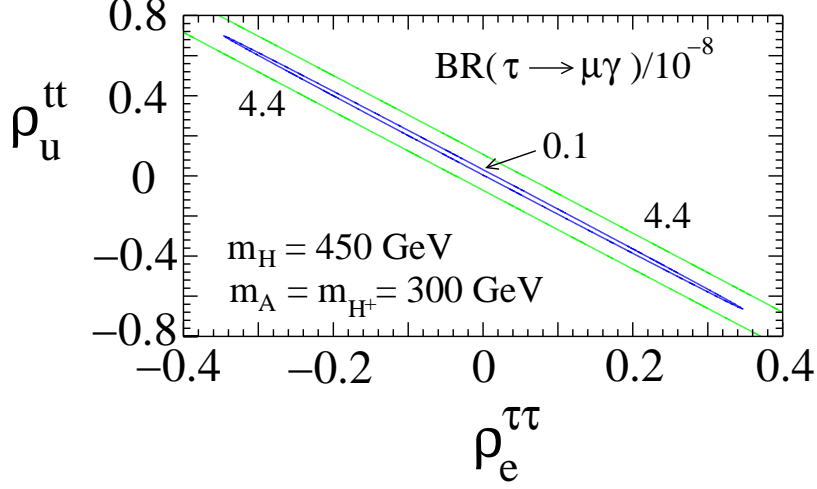


FIG. 3: Branching ratio of $\tau \rightarrow \mu\gamma$ is shown as a function of $\rho_e^{\tau\tau}$ and ρ_u^{tt} . It is assumed that $m_H = 450$ GeV and $m_A = m_{H^+} = 300$ GeV, $s_{\beta\alpha} = 0.9999$ and $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ whose values are determined to realize $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$. The lines for $\text{BR}(\tau \rightarrow \mu\gamma) = 4.4 \times 10^{-8}$ (current limit) and 1×10^{-9} are shown. For this parameter set, the predicted value of δa_μ is 2.1×10^{-9} .

process. The amplitude of $\tau \rightarrow \mu\gamma$ process is parametrized by

$$T = e\epsilon^{\alpha*}\bar{u}_\mu m_\tau i\sigma_{\alpha\beta} q^\beta (A_L P_L + A_R P_R) u_\tau, \quad (9)$$

where $P_{R,L} = (1 \pm \gamma_5)/2$ are chirality projection operators, and e , ϵ^α , q and u_f are the electric charge, a photon polarization vector, a photon momentum, and a spinor of the fermion f , respectively. The branching ratio is given by

$$\frac{\text{BR}(\tau \rightarrow \mu\gamma)}{\text{BR}(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau)} = \frac{48\pi^3\alpha (|A_L|^2 + |A_R|^2)}{G_F^2}, \quad (10)$$

where α and G_F are the fine structure constant and Fermi constant, respectively. The lepton-flavor-violating Higgs contributions to A_L and A_R are given by

$$A_{L,R} = \sum_{\phi=h,H,A,H^-} A_{L,R}^\phi, \quad (11)$$

$$\begin{aligned} A_L^\phi &= \frac{y_{\phi\tau\tau} y_{\phi\tau\mu}}{16\pi^2 m_\phi^2} \left(\log \frac{m_\phi^2}{m_\tau^2} - \frac{4}{3} \right), \quad (\phi = h, H) \\ A_L^A &= \frac{y_{A\tau\tau} y_{A\tau\mu}}{16\pi^2 m_A^2} \left(\log \frac{m_A^2}{m_\tau^2} - \frac{5}{3} \right), \\ A_R^\phi &= A_L^\phi |_{y_{\phi\tau\mu} \rightarrow y_{\phi\mu\tau}}, \quad (\phi = h, H, A), \\ A_L^{H^-} &= -\frac{(\rho_e^\dagger \rho_e)^{\mu\tau}}{192\pi^2 m_{H^-}^2}, \quad A_R^{H^-} = 0, \end{aligned} \quad (12)$$

where $A_{L,R}^\phi$ ($\phi = h, H, A, H^-$) are the ϕ contributions at the one loop level. We also include Barr-Zee-type two-loop contributions to $A_{R,L}$ in the numerical analysis, as studied in Refs. [5, 25, 26].⁵ When we assume non-zero $\rho_e^{\mu\tau(\tau\mu)}$

⁵ We have found a disagreement between our expression of the one loop contributions $A_{R,L}^\phi$ and one given in Refs. [5, 25], and our relative sign between the one and two loop contributions differs from one in Refs. [5, 25].

as suggested by the CMS excess in $h \rightarrow \tau\mu$, but other ρ_f couplings are negligibly small, the predicted branching ratio of $\tau \rightarrow \mu\gamma$ is smaller than the current experimental limit ($\text{BR}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8}$ at the 90% CL. [27, 28]), however, it would be within a reach of the future B-factory. If unknown Yukawa couplings ρ_f other than $\rho_e^{\mu\tau(\tau\mu)}$ are non-zero, the branching ratio can be significantly increased.

Fig. 3 shows the branching ratio of $\tau \rightarrow \mu\gamma$ as functions of $\rho_e^{\tau\tau}$ and ρ_u^{tt} in the presence of the non-zero $\rho_e^{\mu\tau(\tau\mu)}$. Note that ρ_u^{tt} appears in the Barr-Zee diagrams. Here we have assumed that other ρ_f Yukawa couplings are negligible, and $m_H = 450$ GeV, $m_A = m_{H^\pm} = 300$ GeV and $s_{\beta\alpha} = 0.9999$ are given. We choose $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ to achieve the positive contribution to δa_μ and the values of $\rho_e^{\mu\tau(\tau\mu)}$ are determined to explain the CMS excess, $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$. In Fig. 3, the line for the current experimental limit $\text{BR}(\tau \rightarrow \mu\gamma) = 4.4 \times 10^{-8}$ [27, 28] is shown. One sees that the limit strongly constrains $\rho_e^{\tau\tau}$ and ρ_u^{tt} , however, they can still be of $O(1)$ if the signs of them are opposite, which is due to a cancellation between the one- and two-loop contributions. The line for a future reference $\text{BR}(\tau \rightarrow \mu\gamma) = 1 \times 10^{-9}$ [29] is also shown. As one can see from Fig. 3, even if $\rho_u^{tt} = \rho_e^{\tau\tau} = 0$ is satisfied, the branching ratio can be as large as 10^{-9} . The future improvement on the search for $\tau \rightarrow \mu\gamma$ at the level of 10^{-9} will be crucial to test this scenario. In passing, the nonzero ρ_u^{tt} can contribute to δa_μ via the Barr-Zee diagrams. However, it is found that its effect is subdominant.

For other tau decay modes [30], non-zero $\rho_e^{\mu\tau(\tau\mu)}$ couplings induce a correction to $\tau \rightarrow \mu\nu\bar{\nu}$ mode. We find that the correction is of $O(10^{-5} - 10^{-3})$ for the parameter space where the muon g-2 can be explained, and it is consistent with the current experimental results. For $\tau \rightarrow \mu ll$ ($l = \mu, e$), the non-zero branching ratios are predicted even if only $\rho_e^{\mu\tau(\tau\mu)}$ are non-zero. The predicted rate, however, is well below the current experimental limit. The rate strongly depends on ρ_e^{ll} , and the current limit is setting a strong constraint as $\rho_e^{ll} \lesssim 0.01$ for the parameter set studied in Fig. 3. The future improvement of the sensitivity will be very important.⁶

A general 2HDM may be also responsible for discrepancies in $B \rightarrow D\tau\nu$, $B \rightarrow D^*\tau\nu$ and $B \rightarrow \tau\nu$ processes as studied in Ref. [31]. The couplings $\rho_e^{\mu\tau(\tau\mu)}$ can contribute to $B \rightarrow D\tau\nu$, $B \rightarrow D^*\tau\nu$ and $B \rightarrow \tau\nu$ via a charged Higgs mediation if Yukawa couplings ρ_u relevant to these processes are sizable. However, since the sizable contribution to the muon g-2 requires $\rho_e^{\mu\tau} \sim \rho_e^{\tau\mu}$, they also induce the significant contributions to $B \rightarrow D\mu\nu$, $B \rightarrow D^*\mu\nu$ and $B \rightarrow \mu\nu$ processes, so that it would be difficult to explain these discrepancies, and the relevant Yukawa couplings $\rho_{u(d)}$ should be negligible in our scenario.

In order to explain the muon g-2 anomaly, the relatively light extra Higgs bosons A , H , and H^\pm are required. They will be expected to be produced at the LHC experiment. The production via quark Yukawa couplings $\rho_{u,d}$ will be possible and important. Furthermore, in the presence of the sizable ρ_u^{tt} , the gluon fusion production process for A and H would be dominant. However, it is difficult to predict the production cross section without the detail knowledge of the Yukawa couplings $\rho_{e,u,d}$. On the other hand, the production via weak interaction such as $q\bar{q}' \rightarrow W^{\pm*} \rightarrow AH^\pm$ is less model-dependent as discussed in Ref. [32]. The current LHC experimental data would put constraints on various unknown Yukawa couplings ρ_f . The detail study will be worth probing this scenario and we will report it in a forthcoming paper [30].

In conclusion, the CMS experiment has reported the excess in $h \rightarrow \mu\tau$. Although the definitive statement cannot be made until the statistical significance of this excess becomes higher and the ATLAS collaboration also confirms it, this might be a hint for new physics. The general 2HDM can easily accommodate the excess, which can be induced by the $\mu - \tau$ lepton-flavor-violating couplings. We have found that the $\mu - \tau$ flavor violation can significantly enhance the neutral Higgs contributions to the muon g-2, and hence it can explain the anomaly of the muon g-2. In the parameter region where both anomalies for $h \rightarrow \mu\tau$ and the muon g-2 can be solved, the branching ratio of $\tau \rightarrow \mu\gamma$ can be sizable and the search at the future B factory would be crucial to test this scenario. Since the flavor structure of new Yukawa couplings $\rho_{e,u,d}$ is unknown, the further experimental and theoretical studies would be important to reveal the scenario. This will be just a beginning of many of new phenomena beyond the SM.

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⁶ The other flavor violating τ decay such as $\tau \rightarrow \mu\eta$ may also give stringent constraints on the Yukawa couplings in quark sector. For instance, we estimate the upper bound on ρ_d^{ss} from the bound on $\text{BR}(\tau \rightarrow \mu\eta)$: $\rho_d^{ss} \lesssim 0.01$ in the case with $m_A = 350$ GeV. This is almost the same order as the one of $\rho_e^{\mu\mu}$.

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